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Generalized Double Difference Sequence Spaces Defined by Orlicz Functions

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Key words :

Difference sequence, Entire sequence, Analytic sequence, Orlicz function.

Abstract

The idea of difference sequence spaces was introduced by Kizmaz [1] and then this subject has been studied and generalized by various mathematicians. In this paper we define some difference sequence spaces by Orlicz space of entire sequences and establish some inclusion relations. Some properties of these spaces are studied.

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Introduction

A complex sequence, whose k[th] term is x_k is denoted by $\{x_{ij}\}$ or simply. Let ? be the set of all finits sequences. A sequence x = $\{x_{ij}\}$ is said to be analytic if $\sup_{ij} |x_{ij}|^{1/i+j} < \infty$. The vector space of all analytic sequences will be denoted by $\wedge^2$. A sequence x is called entire sequence is $\lim_{i+j\to\infty} |x_{ij}|^{1/i+j}$ = 0. The vector space of all entire sequences will be denoted by Γ^2. Throughout the article $\Gamma_M^2 . \wedge_M^2$ denote the orlicz space entire and analytic sequences respectively.

Throughout m,n denotes an arbitrary positive integer. Kizmaz [1] introduced the notation of difference sequence spaces as follows : X(?) {x = (x_{ij}) : (Δx_{ij})?X}; for X = λ_∞^2, c^2, c_0^2, where ?x = (Δx_{ij}) = $(x_{ij} - x_{i+1,j+1})$. Later on the notion was generalized by Et and Colak [2] as follows: X(Δ_n^m) = {x = (x_{ij}) : (Δ_n^m x_{ij}) ? X} for X= λ_∞^2, c^2, c_0^2, where m?N, Δ_n^m x=(x_{ij}) and Δ_n^m x=(Δ_n^m x_{ij})=($\Delta^{m-1} x_{ij}$? $\Delta^{m-1} x_{i+1,j+1}$).

$$\sum_{2\le m+n\le u+v} (-1)_u^v \binom{m}{v}\binom{u}{v}$$ for all i,j ? N

Later on difference sequence spaces have been studied by Et [3], Et and Nuray 4], Colak et al [5], Isik [6], Altin and Et [7] and many others.

Orlicz [8] used the idea of Orlicz function to construct the space (L^M), Lindenstrauss and Tafriri [9] investigated Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space λ_M^2 contains a subspace isomorphic to λ_p^2 ($1 \leq p \leq \infty$). Subsequently different classes of sequence spaces defined by Parashar and Choudhary [10]. Mursaleen et al [11], Bektas and Altin [12], Tripathy et al [13]. Rao and Subramanian [14] and many others. The Orlicz sequencespaces are the special cases of Orlicz spaces studied in Ref [15].

Recall ([18].[15]) an Orlicz function is a function $M : [0, \infty) \to [0, \infty)$ which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$, for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$. If convexity of Orlicz function M is replaced by $M(x+y) \leq M(x)+M(y)$, then this function is called modulus function, defined and discussed by Ruckle [16] and Maddox [17].

Let (Ω, Σ, μ) be a finite measure space. We denote by $E(\mu)$ the space of all (equivalence classes of) Σ-measurable functions x from Ω into $[0, \infty)$. Given an Orlicz function M, we define on $E(\mu)$ a convex functional I_M by

$$I_M(x) = \int_\Omega M(x(t))\, d\mu,$$

and an Orlicz space $L^M(\mu)$ by $L^M(\mu) = \{x \in E(\mu) : I_M(\lambda x) < +\infty$ for some $\lambda > 0\}$, (For detail see [8], [15]).

Lindenstrauss and Tzafriri [9] used the idea of Orlicz function to construct Orlicz sequence space

$$\lambda_M = \left\{x \in w : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{P}\right) < \infty, \text{ for some } p > 0\right\}$$

where w = {all complex sequences}.

The space λ_M with the norm

$$||x|| = \inf\left\{p > 0 : \sum_{k=1}^{\infty} M\left(\frac{|x_k|}{P}\right) \leq 1\right\},$$

Becomes a Banach space which is called an Orlicz sequence space. For $M(t) = t^p$, $1 \leq p < \infty$, the space coincide with the classical sequence space λ_p.

Given a sequence $x = \{x_k\}$ its n^{th} section is the sequence

$x^{(n)} = (x_1, x_2, \ldots, x_n, 0, 0, \ldots)$

$\delta^{(n)} = (0, 0, \ldots, 1, 0, 0, \ldots)$, 1 in the n^{th} place and zero's else where; An FK-space (Frechet coordinate space) is a Frechet space which is made up of numerical sequences and has the property that the coordinate functional $p_k(x) = x_k$ ($k = 1, 2, \ldots$) are continuous.

An FK-space or a metric space X is said to have AK-property if ($\delta^{(n)}$) is a Schauder basis for X or equivalently $x^{(n)} \to x$ (AK stands for Abschnitts Konvergenz or sectional convergence). The space is said to have AD (or be an AD space) if ϕ is dense in X.

We note that AK implies AD by [18].

If X is a sequence space, we define

(i) $X^{©}$ = the continuous dual of X.

(ii) $^2X^{\alpha} = \{a = (a_{ij}) : \sum\sum_{2 \leq i+j \leq N} |a_{ij}\lambda_{ij}x_{ij}| < \infty$, for each $x \in X\}$;

(iii) $^2X^{\beta} = \{a = (a_{ij}) : \sum\sum_{2 \leq i+j \leq N} a_{ij}\lambda_{ij}x_{ij}$ is convergent for each $x \in X\}$;

(iv) $^2X^{\gamma} = \{a = (a_{ij}) : \sup_{m,n} \left|\sum\sum_{2 \leq i+j \leq N} a_{ij}\lambda_{ij}x_{ij}\right| < \infty$, for each $x \in X\}$;

(v) Let X be an FK-space and X ⊃ ?. Then X^f = {f(?$^{(n)}$) : ??X©}, ${}^2X^\alpha$, ${}^2X^\beta$, ${}^2X^\gamma$, are called the ??, ?? and ?? dual of X, respectively.

Note that ${}^2X^\alpha \subset {}^2X^\beta \subset {}^2X^\gamma$. If X ⊂ Y then ${}^2Y^\mu \subset {}^2X^\mu$, for ? = ?, ?, or ?.

Lemma 1.1. (See (9) Theorem 7.2.7)). Let X be an FK-space and X ⊃ ?. Then

(i) X? ⊂ X^f,

(ii) If X has AK, ${}^2X^\beta = {}^2X^f$,

(iii) If X has AD, ${}^2X^\beta$ = ²X?,

We note that ${}^2\Gamma^\alpha = {}^2\Gamma^\beta = {}^2\Gamma^\gamma = \wedge^2$.

Definition 1.2 : The space consisting of all those sequence x in w such that $M\left(\frac{|\lambda_{ij}x_{ij}|}{\rho}\right)^{1/i+j}$?0 as i+j? ∞ for some arbitrary fixed ? > 0 is denoted by Γ_M^2, M being on Orlicz function. In other words $\left\{M\left(\frac{|\lambda_{ij}x_{ij}|}{\rho}\right)^{1/i+j}\right\}$ is a mill sequence. Γ_M^2 is called the Orlicz space of entire sequences. The space Γ_M^2 is a metric space with the metric d(x, y) = $\sup_{i,j} M\left(\frac{|\lambda_{ij}x_{ij} - \lambda_{ij}y_{ij}|^{1/i+j}}{\rho}\right)$ for all x = {x_{ij}} and y ={y_{ij}} in Γ_M^2.

Definition 1.3. If M is a convex function and M(0) = 0, then $M^2(\eta x) \leq \eta M^2(x)$ for all η with $0 < \eta < 1$.

Definition 1.4. The space consisting of all those sequence x in w such that $\left(\sup_{i,j}\left(M\left(\frac{|\lambda_{ij}x_{ij}|}{\rho}\right)^{1/i+j}\right)\right) < \infty$ for some arbitrarily fixed ? > 0 is denoted by $\wedge^2$ M, M² being an Orlicz function. In other words $\left(M\left(\frac{|\lambda_{ij}x_{ij}|^{1/i+j}}{\rho}\right)\right)$ is a bounded sequences. $\wedge^2$ M is called the Orlicz space of bounded sequence.

Definition 1.5. A sequence space E is said to be solid or normal if (?$_{ij}x_{ij}$)?E whenever (xij)?E² and for all sequences of scalars (?$_{ij}$) with |?$_{ij}$| ≤ 1, [20].

Let p = (p_{ij}) be a sequence of positive real numbers with 0 < p_{ij} <sup p_{ij} = G and let D = Max(1.2$^{G?1}$). The for $a_{ij}b_{ij}$?C, the set of complex numbers for all i,j??, we have

$$|a_{ij} + b_{ij}|^{p_{ij}} \leq D|a_{ij}|^{p_{ij}} + |b_{ij}|^{p_{ij}} \qquad ...(1)$$

In this paper, we define the following sequence spaces.

Let M be an Orlicz function, x be locally convex Hausdroff topological linear space whose topology is determined by a set Q of continuous semi-norms q. The symbol $\wedge^2$ (X), Γ^2 (X) denotes the space of all analytic and entire double sequences defined over X. We define the following sequence spaces:

$$\wedge_M^2(\Delta_n^m, p, q, \lambda) = \left\{ x \in \wedge^2(X) : \sup_{m,n} \frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left(m\left(q\left(\frac{\Delta_n^m \lambda_{ij} x_{ij}}{\rho} \right)^{1/i+j} \right) \right)^{p_{ij}} > \infty, \text{ for some } \rho > 0 \right\}$$

$$\Gamma_M^2(\Delta_n^m, p, q, \lambda) =$$

$$\left\{ x \in \Gamma^2(X) : \frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left(m\left(q\left(\frac{\Delta_n^m \lambda_{ij} x_{ij}}{\rho} \right)^{1/i+j} \right) \right)^{p_{ij}} \to 0, \text{as } m+n \to \infty \text{ for some } \rho > 0 \right\}.$$

2. Main Results

In this section we examine some topological properties of spaces $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ and $\wedge_M^2(\Delta_n^m, p, q, \lambda)$ and investigate some inclusion relations between these spaces.

Proposition 2.1. If M is an Orlicz function, then $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ is a linear set over the set of complex numbers C.

Proof. Let x,y� $\Gamma_M(\Delta_n^m, p, q, \lambda)$ and �����C². In order to prove the result, we need to find some �₃ such that

$$\frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left[M\left(q\left(\frac{\Delta_n^m(\alpha\lambda_{ij}x_{ij} + \beta\lambda_{ij}y_{ij})}{\rho_3} \right)^{1/i+j} \right) \right]^{p_{ij}} \text{�0 as m+n��.} \quad (2.1)$$

Since x,y� $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$, there exist some positive �₁ and �₂ such that

$$\frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left[M\left(q\left(\frac{(\Delta_n^m \lambda_{ij} x_{ij})}{\rho_1} \right)^{1/i+j} \right) \right]^{p_{ij}} \text{�0 as m+n��}$$

and

$$\frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left[M\left(q\left(\frac{(\Delta_n^m \lambda_{ij} y_{ij})}{\rho_2} \right)^{1/i+j} \right) \right]^{p_{ij}} \text{�0 as m+n��}$$

Since M is a non-decreasing modules function, q is semi-norm and Δ_n^m is linear then

$$\frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left[M\left(q\left(\frac{\Delta_n^m(\alpha\lambda_{ij}x_{ij} + \beta\lambda_{ij}y_{ij})}{\rho_3} \right)^{1/i+j} \right) \right]^{p_{ij}} \text{�}$$

$$\frac{1}{m+n} \sum_{2 \le i+j \le m+n}\sum \left[M\left(q\left(\frac{(\alpha)^{1/i+j}(\Delta_n^m \lambda_{ij} x_{ij})^{1/i+j}}{\rho_3} + \frac{(\beta)^{1/i+j}(\Delta_n^m \lambda_{ij} y_{ij})^{1/i+j}}{\rho_3} \right)^{1/i+j} \right) \right]^{p_{ij}}$$

$$\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{\Delta_n^m(\alpha\lambda_{ij}x_{ij}+\beta\lambda_{ij}y_{ij})}{\rho_3}\right)^{1/i+j}\right)\right]^{p_{ij}} \text{ □□}$$

$$\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{(\alpha)(\Delta_n^m\lambda_{ij}x_{ij})^{1/i+j}}{\rho_3}+\frac{(\beta)(\Delta_n^m\lambda_{ij}y_{ij})^{1/i+j}}{\rho_3}\right)^{1/i+j}\right)\right]^{p_{ij}}$$

Take □₃ such that $\frac{1}{\rho_3}$ = min $\left\{\frac{1}{|\alpha|}\frac{1}{\rho_1},\frac{1}{|\beta|}\frac{1}{\rho_2}\right\}$

$$\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{\Delta_n^m(\alpha\lambda_{ij}x_{ij}+\beta\lambda_{ij}y_{ij})}{\rho_3}\right)^{1/i+j}\right)\right]^{p_{ij}} \text{ □□}$$

$$\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}x_{ij})^{1/i+j}}{\rho_1}+\frac{(\Delta_n^m\lambda_{ij}y_{ij})^{1/i+j}}{\rho_2}\right)^{1/i+j}\right)\right]^{p_{ij}}$$

$$\text{□□}\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}x_{ij})^{1/i+j}}{\rho_1}\right)\right)^{p_{ij}}+M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}y_{ij})^{1/i+j}}{\rho_2}\right)^{1/i+j}\right)\right]^{p_{ij}}$$

$$\text{□□D}\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}x_{ij})^{1/i+j}}{\rho_1}\right)\right)^{p_{ij}}\right]^{p_{ij}}+\left[M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}y_{ij})^{1/i+j}}{\rho_2}\right)\right)^{p_{ij}}\right]^{p_{ij}}$$

□0 (m+n□□)

By (2.2) and (2.3). Hence $\frac{1}{m+n}\sum\sum_{2\le i+j\le m+n}\left[M\left(q\left(\frac{(\alpha\Delta_n^m\lambda_{ij}x_{ij}+\beta\Delta_n^m\lambda_{ij}y_{ij})}{\rho_3}\right)^{1/i+j}\right)\right]^{p_{ij}}$ □0 as m+n□□. So (□x + □y)□ $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$. Therefore $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ is a linear space. This completes the proof.

Proposition 2.2. $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ are paranormed spaces (not totally paranormed) with

$$g_\Delta^2{}^*(x) = \inf\left\{\rho^{p_{mn}/H} : \sup_{i,j\ge1}\left[M\left(q\left(\frac{(\Delta_n^m\lambda_{ij}x_{ij})^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}\le 1, \rho>0\right\}, \text{ where } H=\max\left(1,\sup_{i,h} p_{ij}\right)$$

Proof. Clearly g_Δ^2 (x) □□0, g_Δ^2 (x) = g_Δ^2 (□x) and $g_\Delta(\theta)=0$, where □ is the zero sequence of X.

Let (x_{ij}), (y_{ij})□ $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$. Let □₁ and □₂ > 0 be such that

$$\sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}} \le 1 \text{ and } \sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho_2}\right)\right)\right]^{p_{ij}} \le 1.$$

Then

$$\sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\left(\lambda_{ij}x_{ij}+\lambda_{ij}y_{ij}\right)\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \le \left(\frac{\rho_1}{\rho_1+\rho_2}\right)\sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}} +$$

$$\left(\frac{\rho_1}{\rho_1+\rho_2}\right)\sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho_2}\right)\right)\right]^{p_{ij}} \le 1.$$

Hence,

$$g_\Delta^2(x+y) \le \inf\left\{\left(\rho_1+\rho_2\right)^{p_{mn}/H} : m,n\in N\right\}$$

$$\le \inf\left\{\left(\rho_1\right)^{p_{mn}/H} : \sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}} \le 1, \rho_1>0, m,n\in N\right\}$$

$$+\inf\left\{\left(\rho_2\right)^{p_{mn}/H} : \sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho_2}\right)\right)\right]^{p_{ij}} \le 1, \rho_2>0, m,n\in N\right\}$$

Thus we have $g_\Delta^2(x+y) \le g_\Delta^2(x)+g_\Delta^2(y)$. Hence, satisfies the training inequality. $g_\Delta^2(\eta x) = \inf$

$$\left\{\rho^{p_{mn}/H} : \sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\eta\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \le 1,\ m,n>N\right\}$$

$$= \inf\left\{r(\eta)^{p_{mn}/H} : \sup_{i,j\ge1}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{r}\right)\right)\right]^{p_{ij}} \le 1, r>0, m,n\in N\right\}$$, where $r = \frac{\rho}{|\lambda|}$.

Hence, $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ is a paranormed space.
This completes the proof.

Proposition 2.3. Let M_1^2 and M_2^2 be two Orlicz function.
Then $\Gamma_{M_1}^2(\Delta_n^m, p, q, \lambda) \cap \Gamma_{M_2}^2(\Delta_n^m, p, q, \lambda) \subseteq \Gamma_{M_1+M_2}^2(\Delta_n^m, p, q, \lambda)$

Proof. Let $x \in \Gamma_{M_1}^2(\Delta_n^m, p, q, \lambda) \cap \Gamma_{M_2}^2(\Delta_n^m, p, q, \lambda)$.
Then there exist ρ_1 and ρ_2 such that

$$\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}}$$ □0 as m+n□□. (3.1)

and

$$\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho_2}\right)\right)\right]^{p_{ij}}$$ □0 as m+n□□. ...(3.2)

Let □ = min $\left(\frac{1}{\rho_1},\frac{1}{\rho_2}\right)$. Then we have

$$\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[\Gamma^2_{M_1+M_2}\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$ □□D $$\left[\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[\Gamma^2_{M_1}\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}}\right]$$

$$+\left[\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[\Gamma^2_{M_2}\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_2}\right)\right)\right]^{p_{ij}}\right]$$ □0 as m+n□□

$$\frac{1}{m+n}\sum_{2\le i+j\le m+n}\sum\left[\Gamma^2_{M_1+M_2}\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$ □0 as m+n□□.

Therefore, x □ $\Gamma^2_{M_1+M_2}\ (\Delta_n^m, p, q, \lambda)$.

This completes the proof.

Proposition 2.4. Let m,n□□□□. Then we have the following inclusions.

(i) $\Gamma^2_M\left(\Delta_{n-1}^{m-1}, p, q, \lambda\right) \subseteq \Gamma^2_M\ (\Delta_n^m, p, q, \lambda)$

(ii) $\wedge^2_m\left(\Delta_{n-1}^{m-1}, p, q, \lambda\right) \subseteq \wedge^2_m\ (\Delta_n^m, p, q, \lambda)$

Proof. We prove the case (i) only. The other cases follows in a similar way, Let x □□ $\Gamma^2_M\left(\Delta_{n-1}^{m-1}, p, q, \lambda\right)$. Then we have

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_{n-1}^{m-1}\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$ □0 as m+n□□, for some □□> 0.

Since M is non-decreasing convex function and q is a semi-norm, we have

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$ □□

$$\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_{n-1}^{m-1}\lambda_{ij}x_{ij}-\Delta_{n-1}^{m-1}\lambda_{i+1j+1}x_{i+1j+1}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$

$$\le D\left[\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_{n-1}^{m-1}\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}}+\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_{n-1}^{m-1}\lambda_{i+1,j+1}x_{i+1,j+1}\right)^{1/i+j}}{\rho_1}\right)\right)\right]^{p_{ij}}\right]$$

$\to 0$ as $m'+n'\to\infty$.

Therefore,

$$\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}\to 0 \text{ as } m'+n'\to\infty.$$

Hence, $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$.

This completes the proof.

Proposition 2.5. Let $0 < p_{ij} \le r_{ij}$ and let $\left\{\frac{r_{ij}}{p_{ij}}\right\}$ be bounded. Then $\Gamma_M^2(\Delta_n^m, p, q, \lambda) \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$.

Proof. Let $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$. Then

$$\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}\to 0 \text{ as } m'+n'\to\infty. \quad ...(5.1)$$

Let $t_{i,j} = \frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$ and $\mu_{ij} = \left\{\frac{p_{ij}}{r_{ij}}\right\}$. Since $p_{ij} \le r_{ij}$, we have $0 < \mu \le 1$. Take $0 < \mu < \mu_{ij}$.

Define

$$u_{i,j} = \begin{Bmatrix} t_{ij} & (t_{ij}\ge 1) \\ 0 & (t_{ij}<1)\end{Bmatrix} \text{ and } v_{ij} = \begin{Bmatrix} 0 & (t_{ij}\ge 1) \\ t_{ij} & (t_{ij}<1)\end{Bmatrix} \quad ...(5.2)$$

$t_{ij} = u_{ij} + v_{ij}$, $t_{ij}^{\lambda_{ij}} = u_{ij}^{\lambda_{ij}} + v_{ij}^{\lambda_{ij}}$.

Now it follows that $u_{ij}^{\lambda_{ij}} \le u_{ij} \le t_{ij}$, $v_{ij}^{\lambda_{ij}} \le v_{ij}^{\lambda}$, since $t_{ij}^{\lambda_{ij}} = u_{ij}^{\lambda_{ij}} + v_{ij}^{\lambda_{ij}}$ then $t_{ij}^{\lambda_{ij}} \le t_{ij} + v_{ij}^{\lambda}$.

$$\frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \le \frac{1}{m'+n'}\sum\sum_{2\le i+j\le m'+n'}\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$

$$\leq \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \text{ } \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}}$$

$$\leq \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \leq \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}}$$

But $\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \to 0$ as $m'+n' \to \infty$ (by (5.1)).

Therefore,

$$\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \to 0 \text{ as } m'+n' \to \infty.$$

Hence, $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$. From (5.1), we get $\Gamma_M^2(\Delta_n^m, r, q, \lambda) \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$. This completes the proof.

Proposition 2.6. (a) Let $0<\inf p_{ij} \leq p_{ij} \leq 1$. Then $\Gamma_M^2(\Delta_n^m, p, q, \lambda) \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$

(b) Let $1 \leq p_{ij} \leq \sup p_{ij} < \infty$. Then $\Gamma_M^2(\Delta_n^m, p, q, \lambda) \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$

Proof. (a) Let $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$. Then

$$\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \to 0 \text{ as } m'+n' \to \infty. \quad \text{...(6.1)}$$

Since $0 < \inf p_{ij} \leq p_{ij} \leq 1$.

$$\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \leq \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \quad \text{...(6.2)}$$

From (6.1) and (6.2) it follows that, $x \in \Gamma_M^2(\Delta_n^m, q, \lambda)$. Thus $\Gamma_M^2(\Delta_n^m, p, q, \lambda) \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$. We have tus proven (a).

(b) Let $p_{ij} \geq 1$ for each i,j and $\sup p_{ij} < \infty$ and let $x \in \Gamma_M^2(\Delta_n^m, q, \lambda)$. Then

$$\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \to 0 \text{ as } m'+n' \to \infty$$

Since $1 \leq p_{ij} \leq \sup p_{ij} < \infty$, we have

$$\frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}} \leq \frac{1}{m'+n'} \sum\sum_{2\leq i+j\leq m'+n'} \left[M \left(q \left(\frac{\left(\Delta_n^m \lambda_{ij} x_{ij}\right)^{1/i+j}}{\rho} \right) \right) \right]^{p_{ij}}$$

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \to 0 \text{ as } m'+n'\to\infty. \text{ (by using (6.3)).}$$

Therefore, $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$.

This completes the proof.

Proposition 2.7.

$\Gamma^2 \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$, with the hypothesis that $\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \le |x_{ij}|^{1/i+j}$.

Proof. Let $x \in \Gamma^2$. Then we have the following implications :

$$|x_{ij}|^{1/i+j} \to 0 \text{ as } i+j\to\infty. \qquad \text{...(7.1)}$$

But $\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \le |x_{ij}|^{1/i+j}$, by our assumption, implies that

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \to 0 \text{ as } m'+n'\to\infty \text{ by (7.1)}$$

Then $x \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$ and $\Gamma^2 \subset \Gamma_M^2(\Delta_n^m, p, q, \lambda)$

This completes the proof.

Proposition 2.8. $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ has AK where M is an Orlicz function.

Proof. Let $x=(x_{ij}) \in \Gamma_M^2(\Delta_n^m, p, q, \lambda)$ but then $\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \in \Gamma^2$, and hence

$$\sup_{i+j\ge m+n+1}\left[\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}\right] \to 0 \text{ as } m'+n'\to\infty. \qquad \text{...(8.1)}$$

Take the n^{th} sectional sequence of x, $x(n) = (x_1, x_2, x_3, \ldots, x_n, 0, \ldots)$. By using (8.1),

$$d(x, x^{(m'+n')}) = \sup_{i+j\ge m+n+1}\left[\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}\right] \to 0 \text{ as } m'+n'\to\infty$$

which implies that $x^{(m',n')} \to x$ as $m'+n'\to\infty$, implying that $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ as AK.

This completes the proof.

Proposition 2.9. $\Gamma_M^2(\Delta_n^m, p, q, \lambda)$ is solid.

Proof. Let | |x_{ij}| □ |y_{ij}| and let y = (y_{ij}) □ $\Gamma_M^2\ (\Delta_n^m, p, q, \lambda)$, because M is non-decreasing

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \text{□□} \frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$$

and because y □□ $\Gamma_M^2\ (\Delta_n^m, p, q, \lambda)$

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \text{□□}\, \Gamma^2 .$$

That is,

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \text{□0 as } m'+n' \text{□□ and}$$

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} \text{□0 as } m'+n' \text{□□.}$$

Therefore x = (x_{ij}) □ $\Gamma_M^2\ (\Delta_n^m, p, q, \lambda)$.

This completes the proof.

Proposition 2.10. $\left[\Gamma_M^2\left(\Delta_n^m, p, q, \lambda\right)\right]^{\beta} = \wedge^2$.

Proof. Step 1.

Γ^2 □□ $\Gamma_M^2\ (\Delta_n^m, p, q, \lambda)$ by Proposition 2.7, this implies that $\left[\Gamma_M^2\left(\Delta_n^m, p, q, \lambda\right)\right]^{\beta} = \wedge^2$.

Therefore,

$$\left[\Gamma_M^2\left(\Delta_n^m, p, q, \lambda\right)\right]^{\beta} \text{□}\, \wedge^2 . \qquad \text{...(10.1)}$$

Step 2. Let y □□ $\wedge^2$. Then |y_{ij}| < $T^{i,j}$ for all I,j and for some constant $T^2 > 0$.

Let x □□ $\Gamma_M^2\ (\Delta_n^m, p, q, \lambda)$. Then $\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$ □0 as $m'+n'$ □□.

Hence,

$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}}$ < □ for given □ > 0 for sufficiently large i,j. Take □ = $\frac{1}{2T}$, so that

$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} < \frac{1}{(2T)^{i,j}}$. But then

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}y_{ij}\right)^{1/i+j}}{\rho}\right)\right)\right]^{p_{ij}} < \frac{1}{2^{i+j}}$$, so that

$$\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}y_{ij}\right)}{\rho}\right)\right)\right]^{p_{ij}}$$ converges.

Therefore, $\sum_{2\le i+j\le m'+n'}\sum\left[\frac{1}{m'+n'}\sum_{2\le i+j\le m'+n'}\sum\left[M\left(q\left(\frac{\left(\Delta_n^m\lambda_{ij}x_{ij}y_{ij}\right)}{\rho}\right)\right)\right]^{p_{ij}}\right]$ converges.

Hence, $\sum_{2\le i+j\le m'+n'}\sum$?$_{ij}x_{ij}y_{ij}$ converges, so that y ? $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta$. Thus

$$\wedge^2 \,?\, \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta \qquad \text{...(10.2)}$$

Step 3. From (10.1) and (10.2), we obtain $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \wedge^2$.
This completes the proof.

Proposition 2.11. $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\mu = \wedge^2$ for m = ?, ?, ?, *f*.

Proof. Step 1.
$\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)$ has AK by proposition 2.8. Hence by Lemma 1.1(ii) we get $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^f$. But $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \wedge^2$. Hence,
$\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^f = \wedge^2$. ...(11.1)

Step 2.
Since AK implies AD, hence by Lemm1 1.1(iii) we get $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\gamma$. Therefore, $\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\gamma = \wedge^2$. ...(11.2)

Step 3.
$\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)$ is normal by Proposition 2.9. Hence by [20, proposition 2.7], we get

$$\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\gamma = \wedge^2. \qquad \text{...(11.3)}$$

From (11.1), (11.2) and (11.3), we have

$$\left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\alpha = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\beta = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^\gamma = \left[\Gamma_M^2\left(\Delta_n^m,p,q,\lambda\right)\right]^f = \wedge^2.$$

This completes the proof.

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